On the neighbour-distinguishing index of a graph

Keith Edwards,* Mirko Horňák[†] and Mariusz Woźniak[‡]

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Abstract

A proper edge colouring of a graph G is neighbour-distinguishing provided that it distinguishes adjacent vertices by sets of colours of their incident edges. It is proved that for any planar bipartite graph G with $\Delta(G) \geq 12$ there is a neighbour-distinguishing edge colouring of G using at most $\Delta(G) + 1$ colours. Colourings distinguishing pairs of vertices that satisfy other requirements are also considered.

1 Introduction

Let G be a finite simple graph with no component K_2. Let C be a finite set of colours and let $\varphi : E(G) \to C$ be a proper edge colouring of G. The *colour set* of a vertex $v \in V(G)$ with respect to φ, in symbols $S_\varphi(v)$, is the set of colours of edges incident with v. The colouring φ is *neighbour-distinguishing* if it distinguishes any two adjacent vertices by their colour sets, i.e., $S_\varphi(u) \neq S_\varphi(v)$ whenever $u, v \in V(G)$ and $uv \in E(G)$. Frequently a neighbour-distinguishing edge colouring will be shortened to an *nd-colouring*. The *neighbour-distinguishing index* of the graph G, denoted by $\mathrm{ndi}(G)$, is the smallest number of colours in an nd-colouring of

*Division of Applied Computing, University of Dundee, Dundee, DD1 4HN, United Kingdom, e-mail: `kjedwards@dundee.ac.uk`

[†]Institute of Mathematics, P. J. Šafárik University, Jesenná 5, 040 01 Košice, Slovakia, e-mail: `hornak@science.upjs.sk`

[‡]Faculty of Applied Mathematics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland, e-mail: `mwozniak@uci.agh.edu.pl`