Instytut Informatyki Uniwersytetu Jagiellonskiego w Krakowie
serdecznie zaprasza na wyklad

♣ *The Promise & Challenge of Multidimensional Visualization*

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The fascination with "dimensionality" predates even Aristotle. Since the nineteenth century advances in Science and Mathematics unshackled our imagination with higher-dimensional geometries and multi-dimensional(multivariate) problems. These can now be *visualized* with a system of *Parallel Coordinates*. The *perceptual* barrier imposed by our 3-dimensional habitation has been breached.

We describe how this visualization works and demonstrate some of its applications: in air traffic control (3 patents - collision avoidance), data exploration (patent - example: discovering banks' manipulation of gold market), modeling complex relations (example: interactive visual model of a country's economy), and new representation of surfaces preferable even for some 3-dimensional applications. Results are first discovered *visually* and then proven mathematically; in the true spirit of Geometry. Our 3-dimensional experience is now the laboratory for insights into complex high-dimensional situations.

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A. Inselberg, "Parallel Coordinates: Visual Multidimensional Geometry and its Applications", Springer New York, 2009. This book was praised by ***Stephen Hawking*** among others.

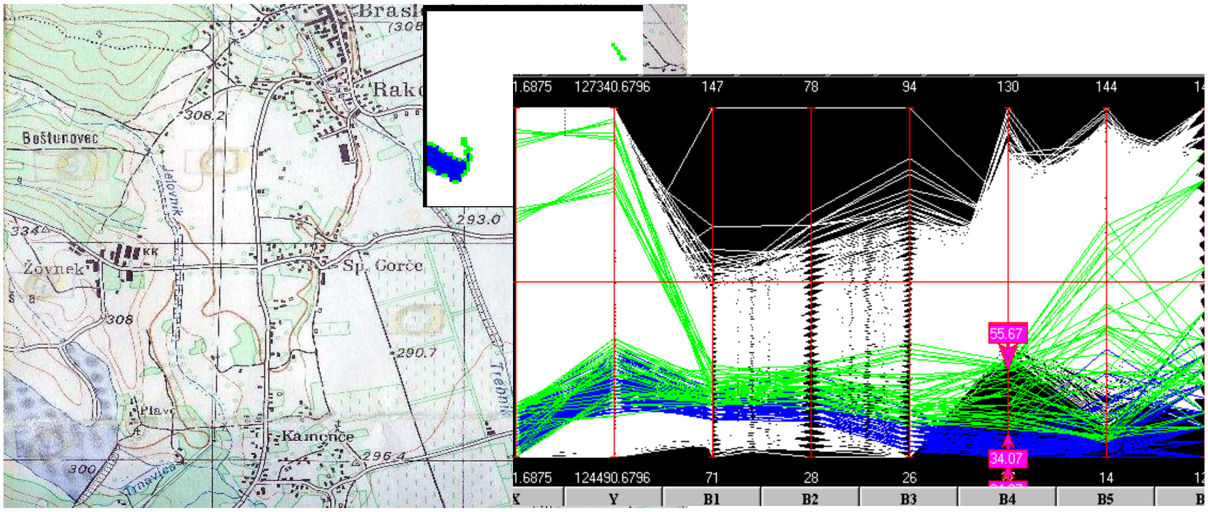

Figure 1: Exploratory Data Analysis, ground emissions measured by satellite on a region of Slovenia, on the left, are displayed on the right. In the middle, water (in blue) and the lake's edge (in green) are discovered by the indicated queries.

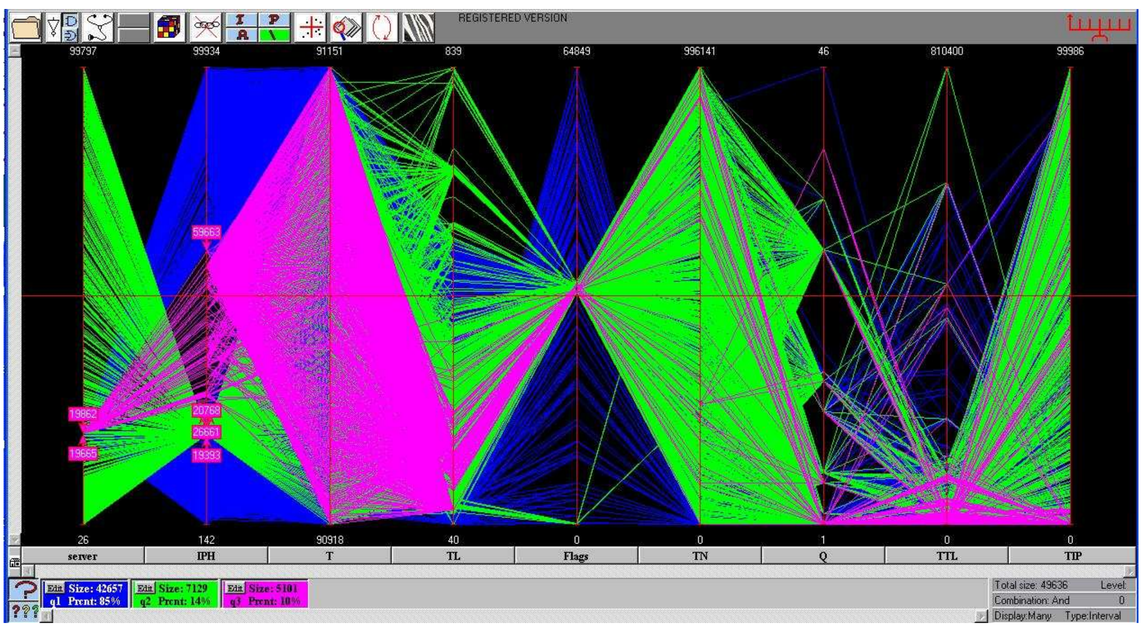

Figure 2: Detecting Network Intrusion from Internet Traffic Flow Data. The server with IP address on the first axis (left) is "bombarding" other servers (IP address on 2nd axis) and starting a chain reaction "botnet".

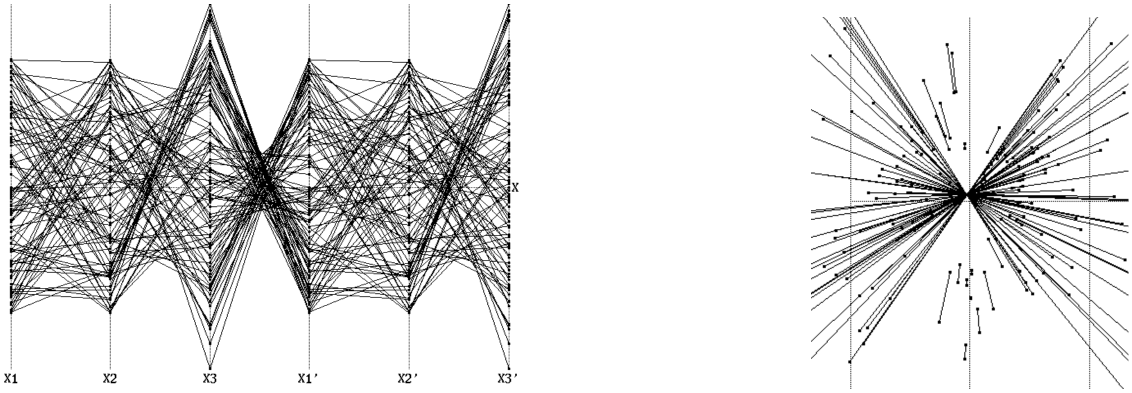

Figure 3: (left) The polygonal lines on the first 3 axes represent a randomly chosen set of coplanar points. There is no discernible pattern. (right) Seeing coplanarity! Two points represent a line on the plane and are determined from the intersection of two polygonal lines. The straight lines joining the pairs of points intersect. That is in ||-coords a plane is not recognized from (the representation of) its points but from (the representation) of its lines (right). The *recursive* visualization generalizes to any dimension.

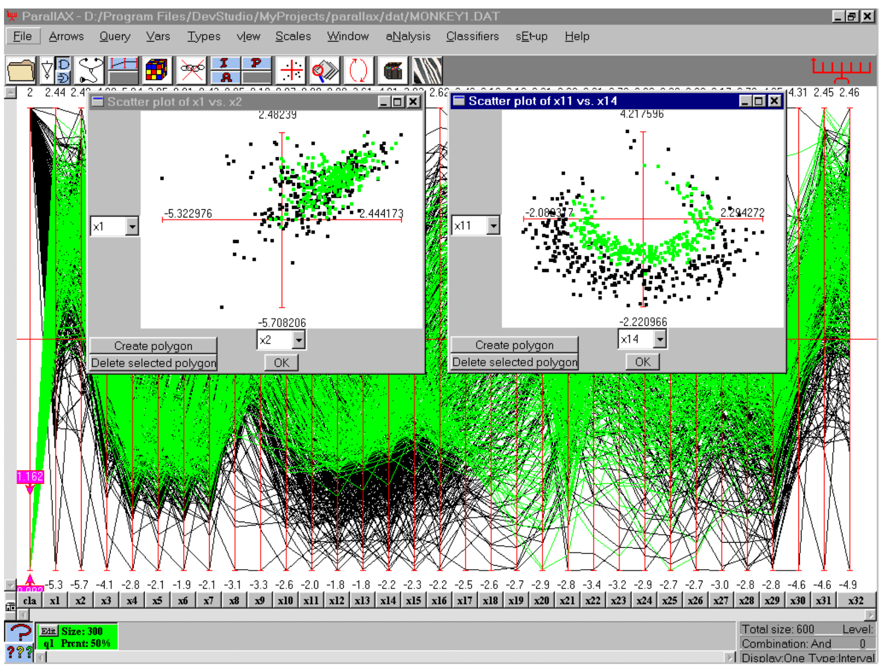

Figure 4: In the background is a dataset with 32 variables and 2 categories. On the left is the plot of the first two variables in the original order and on right the best two variables after classification. The algorithms discovers the best 9 variables (features) needed to describe the rule, with 4% error, and orders them according to their predictive power.

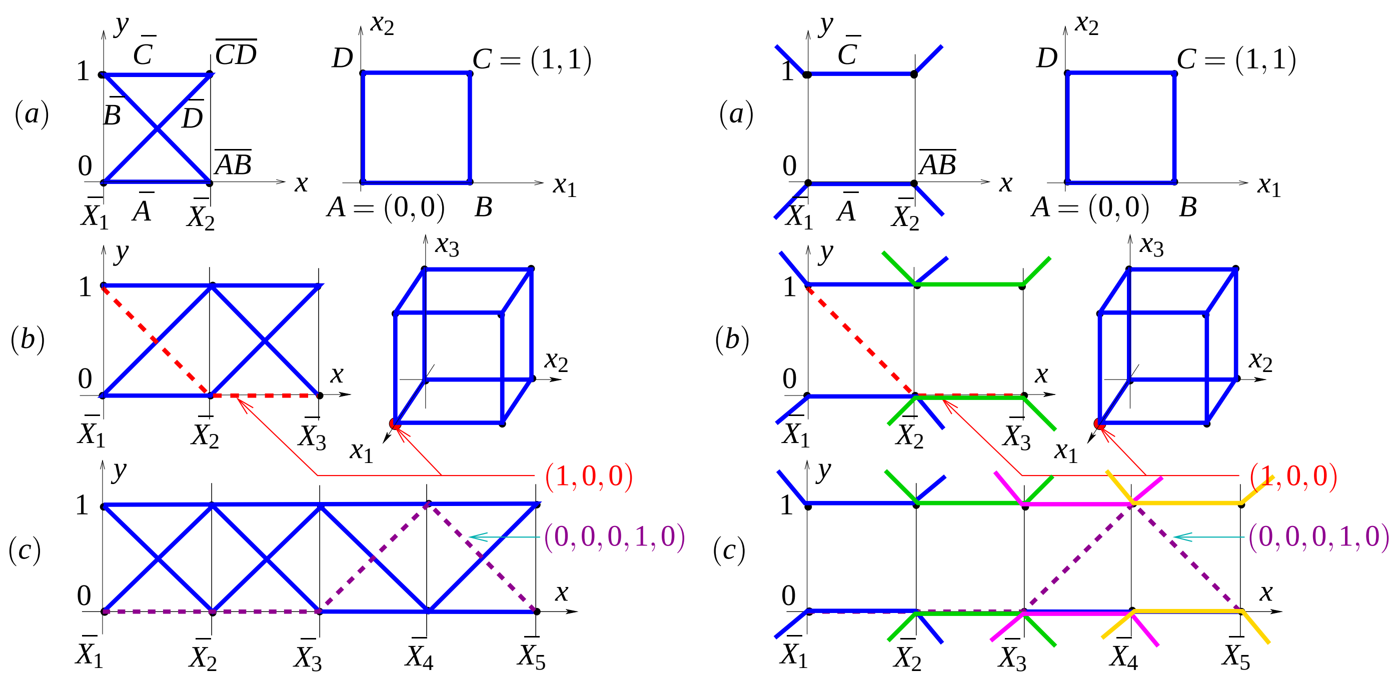

Figure 5: Square, cube and hypercube in 5-D on the left represented by their vertices and on the right by the tangent planes. Note the hyperbola-like (with 2 assymptotes) regions showing that the object is convex.

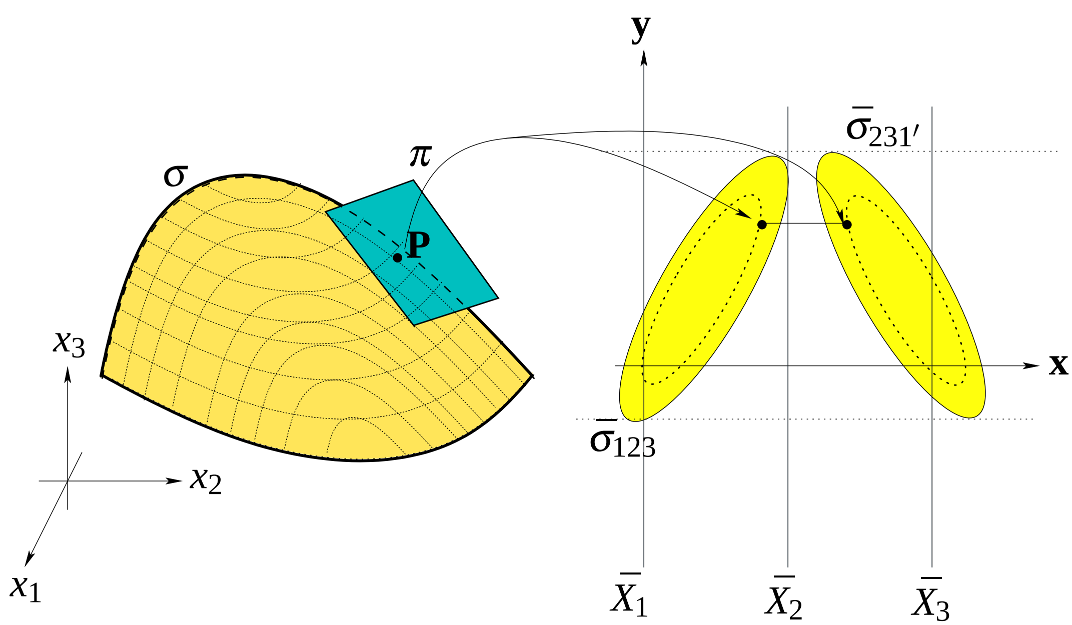

Figure 6: In 3-D a surface σ is represented by two linked planar regions $\bar{\sigma}_{123}$, $\bar{\sigma}_{231'}$. They consist of the pairs of points representing all its tangent planes. In N-dimensions a hypersurface is represented by $(N-1)$ regions as the hypercube above.

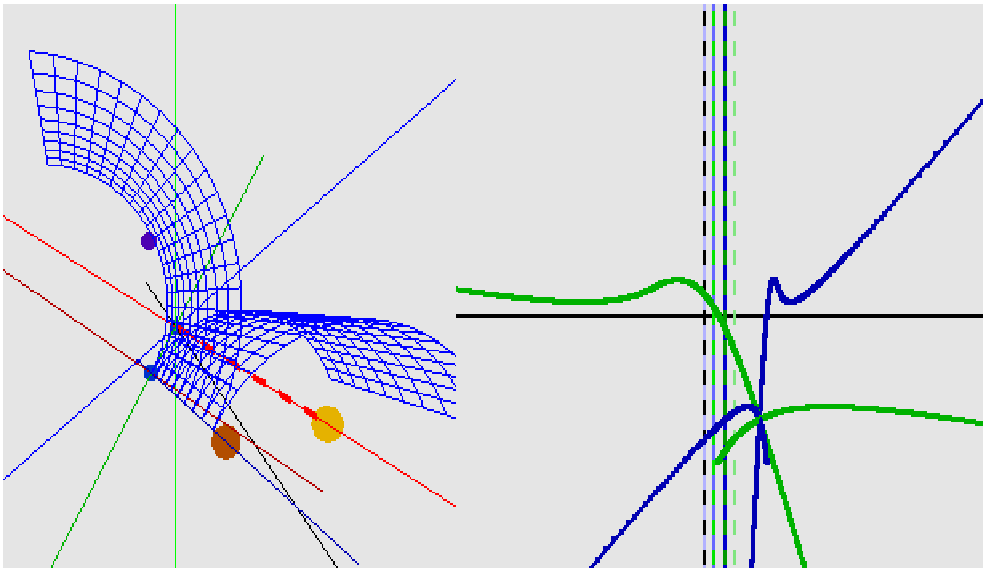

Figure 7: Developable surfaces are represented by curves. Note the two dualities *line of cusps line* $\leftrightarrow$ *inflection points* and *bitangent plane* $\leftrightarrow$ *crossing point*. Three such curves represent the corresponding hypersurface in 4-D and so on.

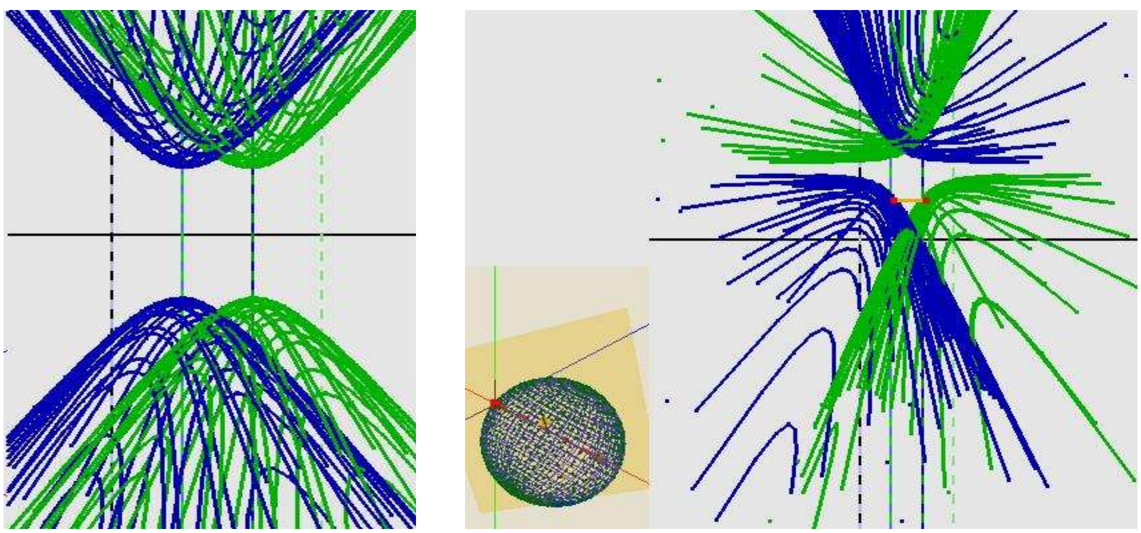

Figure 8: Representation of a sphere centered at the origin (left) and after a translation along the x_1 axis (right) causing the two hyperbolas to rotate in opposite directions. Note the *rotation* $\leftrightarrow$ *translation* duality. In N-D a sphere is represented by $(N-1)$ such hyperbolic regions — pattern repeats as for hypercube above.

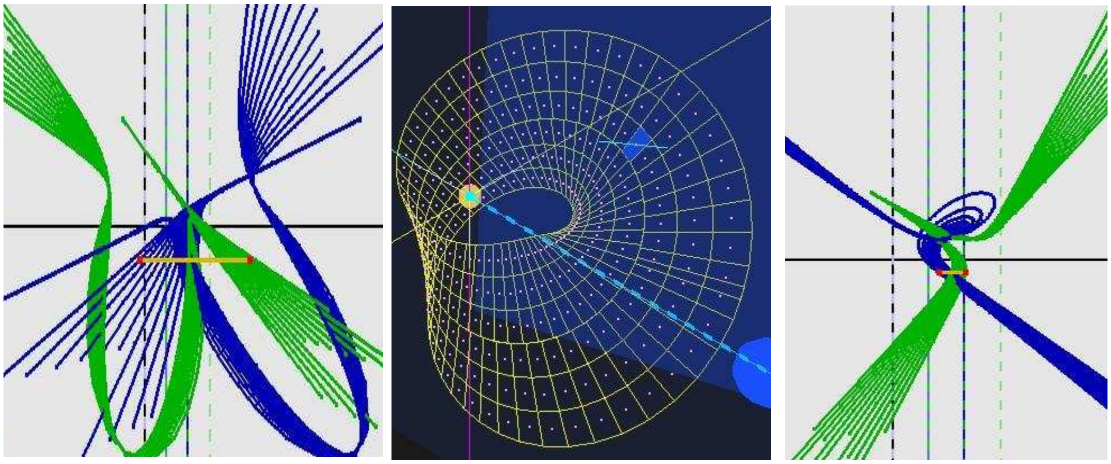

Figure 9: Möbius strip and its representation for two orientations. The two cusps on the left show that it has an “inflection-point in 3-D” (twist)– opposite direction of duality in Fig 7

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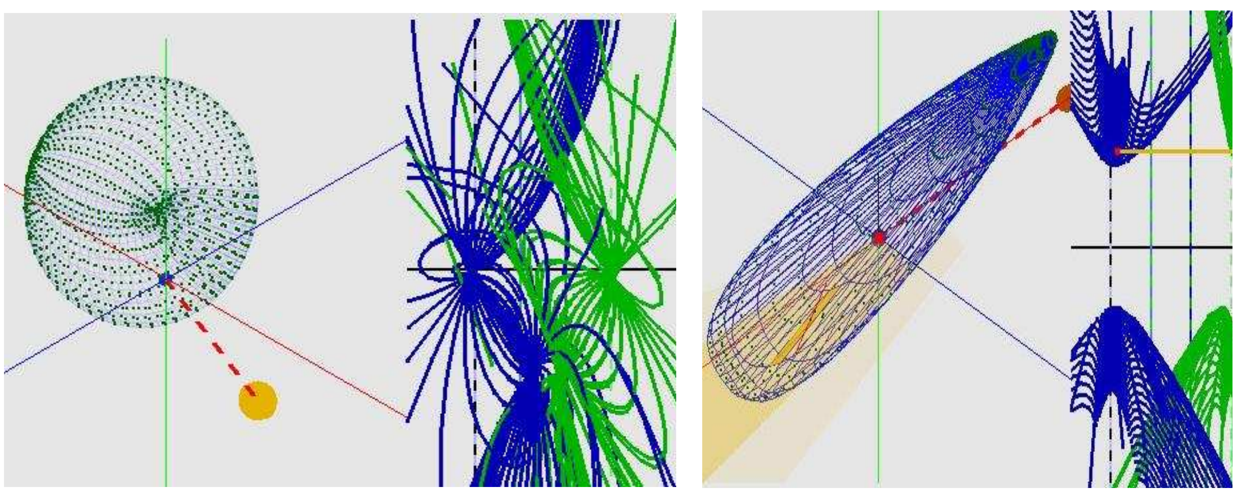

Figure 10: Representation of a surface with 2 “dimples” (depressions with cusp) which are mapped into a pair of “swirls” and are **all** visible. By contrast, in the perspective (left) one dimple is hidden. Convex objects in *any* dimension are represented by hyperbola-like regions. On the right is a convex surface in 3-D and its representation.

Figure 11: Interior point (polygonal line) construction algorithm shown for a convex hypersurface in 20 - D. A polygonal line touching any of the intermediate curves represents a point on the surface, and if it intersects one of the curves it represents an exterior point. This pattern represents a relation, such as in a process, among the 20 variables and the polygonal line a *feasible state*. The narrowest ranges for $X13, X14, X15$ show that these are the critical variables – closest to boundary.