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Preprint Nr MD 075
(otrzymany dnia 16.12.2014)

**Kraków
2014**

Distinguishing graphs by total colourings *

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Abstract

We introduce the *total distinguishing number* $D''(G)$ of a graph G as the least number d such that G has a total colouring (not necessarily proper) with d colours that is only preserved by the trivial automorphism. This is an analog to the notion of the distinguishing number $D(G)$, and the distinguishing index $D'(G)$, which are defined for colourings of vertices and edges, respectively. We obtain a general sharp upper bound: $D''(G) \leq \lceil \sqrt{\Delta(G)} \rceil$ for every connected graph G.

We also introduce the *total distinguishing chromatic number* $\chi''_D(G)$ similarly defined for proper total colourings of a graph G. We prove that $\chi''_D(G) \leq \chi''(G) + 1$ for every connected graph G with the total chromatic number $\chi''(G)$. Moreover, if $\chi''(G) \geq \Delta(G) + 2$, then $\chi''_D(G) = \chi''(G)$. We prove these results by setting sharp upper bounds for the minimal number of colours in a proper total colouring such that each vertex has a distinct set of colour walks emanating from it.

Keywords: total distinguishing number; total distinguishing chromatic number; automorphism; symmetry breaking in graphs
Mathematics Subject Classifications: 05C25, 05C80, 03E10

*The research was partially supported by the Polish Ministry of Science and Higher Education. The third author was supported by the NCN grant DEC-2013/09/B/ST1/01772, and his research was done during his visit in the Institut Mittag-Leffler (Djursholm, Sweden).