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for the Distinguishing Index*

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# Improving Upper Bounds for the Distinguishing Index

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## Abstract

The distinguishing index of a graph  $G$ , denoted by  $D'(G)$ , is the least number of colours in an edge colouring of  $G$  not preserved by any non-trivial automorphism. We characterize all connected graphs  $G$  with  $D'(G) \geq \Delta(G)$ . We show that  $D'(G) \leq 2$  if  $G$  is a traceable graph of order at least seven, and  $D'(G) \leq 3$  if  $G$  is either claw-free or 3-connected and planar. We also investigate the Nordhaus-Gaddum type relation:  $2 \leq D'(G) + D'(\overline{G}) \leq \max\{\Delta(G), \Delta(\overline{G})\} + 2$  and we confirm it for some classes of graphs.

**Keywords:** edge colouring; symmetry breaking in graph; distinguishing index; claw-free graph, planar graph

Mathematics Subject Classifications: 05C05, 05C10, 05C15, 05C45

## 1 Introduction

We follow standard terminology and notation of graph theory (cf. [12]). In this paper, we consider general, i.e. not necessarily proper, edge colourings of graphs. Such a colouring  $f$  of a graph  $G$  *breaks an automorphism*  $\varphi \in \text{Aut}(G)$  if  $\varphi$  does not preserve colours of  $f$ . The *distinguishing index*  $D'(G)$

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